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This detail from Michelangelo's 'The Fall of Man' focuses on the figures of Adam and Eve. Adam is depicted in a reclining position, his body angled away from the viewer but his head turned to look back over his right shoulder. He is draped in a light pinkish-red cloth that covers his lower body. Eve is seated next to him, her body more upright and facing towards the right. She is also draped in a similar pinkish-red cloth. Both figures have a contemplative or perhaps regretful expression. The background is dark and indistinct, emphasizing the figures. The lighting is soft, highlighting the contours of their bodies and the folds of the fabric.

3.5 Short Recapitulation of First Order Logic



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First Order Logic - FOL

quantifier	name	intentional meaning
$\exists$	existential quantifier	“it exists”
$\forall$	universal quantifier	“for all

- **Operators (logical connectives)** as in propositional logic
- **Variables**, e.g., $X, Y, Z, \dots$
- **Constants**, e.g., $a, b, c, \dots$
- **Functions**, e.g., $f, g, h, \dots$ (incl. arity)
- **Relations / Predicates**, e.g., $p, q, r, \dots$ (incl. arity)

$$(\forall X)(\exists Y)((p(X) \vee \neg q(f(X), Y)) \rightarrow r(X))$$

First Order Logic - FOL

FOL: Syntax

- „correct“ formulation of **Terms** from Variables, Constants and Functions:
 - $f(X)$, $g(a, f(Y))$, $s(a)$, $.(H, T)$, $x_location(Pixel)$
- „correct“ formulation of **Atoms** from Relations with **Terms** as arguments
 - $p(f(X))$, $q(s(a), g(a, f(Y)))$, $add(a, s(a), s(a))$,
 $greater_than(x_location(Pixel), 128)$
- „correct“ formulation of **Formulas** from **Atoms**, Operators and Quantifiers:
 - $(\forall Pixel)(greater_than(x_location(Pixel), 128) \rightarrow red(Pixel))$
- If in doubt, use brackets!
- All Variables should be quantified!

First Order Logic - FOL

How to model Facts?

- „All kids love icecream.“
 $\forall X: \text{Child}(X) \rightarrow \text{lovesIcecream}(X)$
- „The father of a person is its male parent.“
 $\forall X \forall Y: \text{isFather}(X,Y) \leftrightarrow (\text{Male}(X) \wedge \text{isParent}(X,Y))$
- „There are (one or more) interesting lectures.“
 $\exists X: \text{Lecture}(X) \wedge \text{Interesting}(X)$
- „The relation ,isNeighbor‘ is symmetric.“
 $\forall X \forall Y: \text{isNeighbor}(X,Y) \rightarrow \text{isNeighbor}(Y,X)$

First Order Logic - FOL

How to model Facts?

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 $\forall X \forall Y: \text{isNeighbor}(X,Y) \rightarrow \text{isNeighbor}(Y,X)$

First Order Logic - FOL

How to model Facts?

- There is a significant difference:

- „All kids love icecream.“

$\forall X: \text{Child}(X) \rightarrow \text{lovesIcecream}(X)$

it is possible that X is not a kid but (nevertheless) loves icecream

$\forall X: \text{Child}(X) \wedge \text{lovesIcecream}(X)$

there are only kids in the universe of discourse and they love icecream

$\forall X: \text{Child}(X)$	$\forall X: \text{lovesIcecream}(x)$	$\forall X: \text{Child}(X) \rightarrow \text{lovesIcecream}(X)$	$\forall X: \text{Child}(X) \wedge \text{lovesIcecream}(X)$
0	0	1	0
0	1	1	0
1	0	0	0
1	1	1	1

First Order Logic - FOL

How to model Facts?

- There is a significant difference:

- „There are (one or more) interesting lectures.“

$\exists X: \text{Lecture}(X) \wedge \text{Interesting}(X)$

the things we address here must have both properties

$\exists X: \text{Lecture}(X) \rightarrow \text{Interesting}(X)$

here X is also qualified, if X is not a lecture

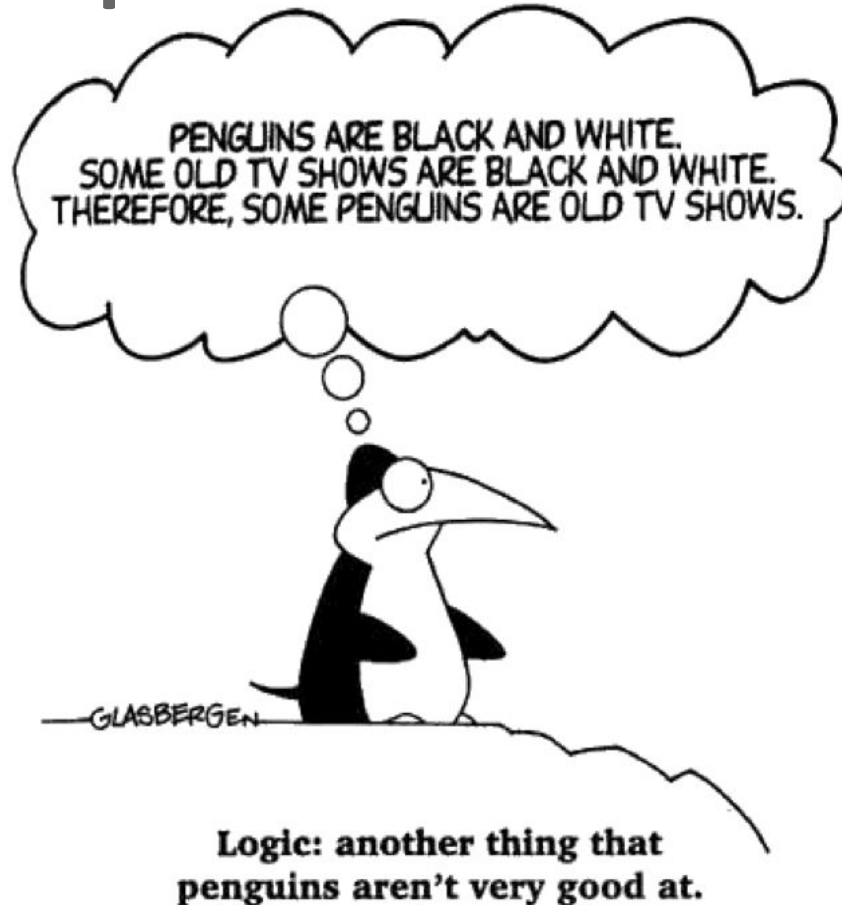
$\exists X: \text{Lecture}(X)$	$\exists X: \text{Interesting}(X)$	$\exists X: \text{Lecture}(X) \rightarrow \text{Interesting}(X)$	$\exists X: \text{Lecture}(X) \wedge \text{Interesting}(X)$
0	0	1	0
0	1	1	0
1	0	0	0
1	1	1	1

First Order Logic - FOL

Model-theoretic Semantics

- **Structure:**
 - Definition of a **domain** D.
 - **Constant symbols** are mapped to elements of D.
 - **Function symbols** are mapped to functions in D.
 - **Relation symbols** are mapped to relations over D.
- **Then:**
 - **Assertions** will become elements of D.
 - **Relation symbols** with arguments will become **true** or **false**.
 - **Logical connectives** and **quantifiers** are treated likewise.

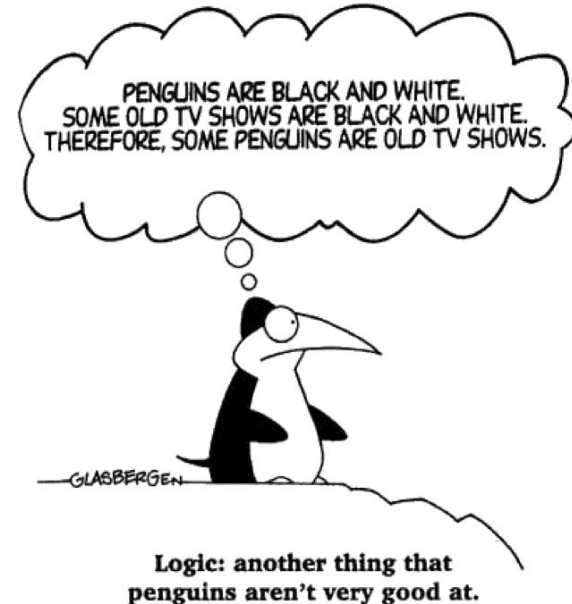
The Penguin Example



The Penguin Example

```
( (∀X)( penguin(X) → blackandwhite(X) )  
∧ (∃X)( oldTVshow(X) ∧ blackandwhite(X)  
)  
 ) → (∃X)( penguin(X) ∧ oldTVshow(X) )
```

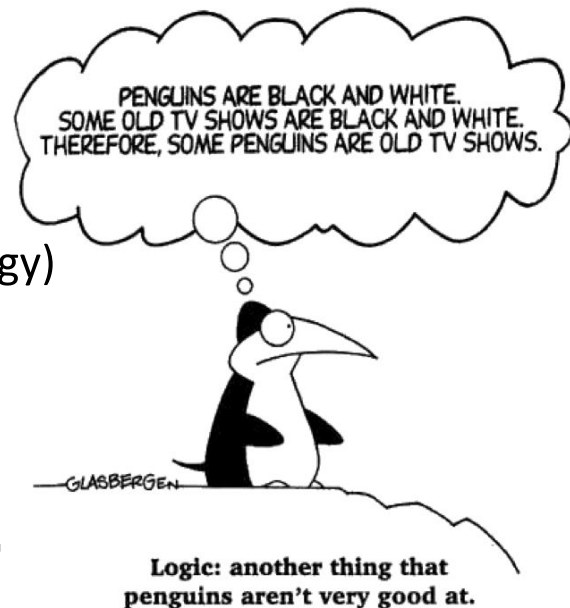
What is the intentional Semantics?



The Penguin Example

$$\begin{aligned} & ((\forall X)(\text{penguin}(X) \rightarrow \text{blackandwhite}(X)) \\ & \wedge (\exists X)(\text{oldTVshow}(X) \wedge \text{blackandwhite}(X)) \\ &) \\ &) \rightarrow (\exists X)(\text{penguin}(X) \wedge \text{oldTVshow}(X)) \end{aligned}$$

- Interpretation $\mathcal{I}$:
 - **Domain:** a set M , containing elements a, b .
 - ... no constants or function symbols ...
 - We show: the formula is *refutable* (i.e. it is not a tautology)
 - If $\mathcal{I}(\text{penguin})(a)$, $\mathcal{I}(\text{blackandwhite})(a)$,
 $\mathcal{I}(\text{oldTVshow})(b)$, $\mathcal{I}(\text{blackandwhite})(b)$ is true ,
 $\mathcal{I}(\text{penguin})(b)$ and $\mathcal{I}(\text{oldTVshow})(a)$ is false,
 - then the formula with Interpretation $\mathcal{I}$ is **false**, i.e. $\mathcal{I} \not\models \mathcal{F}$



First Order Logic - FOL

Logical Entailment

- a **theory** $\mathbb{T}$ is a set of formulas.
- an interpretation $\mathbb{I}$ is a **model** of $\mathbb{T}$, iff $\mathbb{I} \models G$ for all formulas G in $\mathbb{T}$.
- a formula F is a **logical consequence** of $\mathbb{T}$,
iff all models of $\mathbb{T}$ are also models of F .
- then we write $\mathbb{T} \models F$.
- two formulas F, G are called **logically equivalent**,
iff $\{F\} \models G$ and $\{G\} \models F$.
- then we write $F \equiv G$

Theory $\doteq$ Knowledge Base

First Order Logic - FOL

Some Properties of FOL

- **Monotony**

- If the knowledge base grows, all previously possible entailments hold.
- S and T are theories, with $S \subseteq T$
- Then it holds that $\{F \mid S \models F\} \subseteq \{F \mid T \models F\}$

- **Compactness**

- For each entailment made from a theory, a finite subset of the theory is sufficient.

First Order Logic - FOL

Some Properties of FOL

- **Semi-decidability**
 - FOL is **not decidable**
 - But, FOL is **semi-decidable**,
i.e. a logical consequence $\mathcal{T} \models \mathcal{F}$ always can be
proven in finite time (but not necessarily also $\mathcal{T} \not\models \mathcal{F}$)



06: How to Mechanize Reasoning - Tableaux Algorithm

OpenHPI - Course Knowledge Engineering with Semantic Web Technologies

Lecture 3: Ontologies and Logic